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The Maximal Soluble Subgroups of the Group GL(15,p^k)

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ABSTRACT: In this paper we determine the JS -Maximal soluble subgroups of the group $GL(14,p^k)$, and prove that the number types of JS –maximal soluble subgroups in the group $GL(15,p^k)$ is 10 and are $M_i, i=1,...,10$.

***Keywords*:** JS-maximale, Soluable, Transitive

INTRODUCTION

Cole(1895)lists the transitive group of degree 10.Miller(1895a) shows that Cole's list Of imprimitive group has six repetitions.With these corrections' Cole's numbers for the transitive groups agree with those of Butler and Mckay(1983) and Sims(1970).Miller(18'5b) gives a list of the in transitive groups of degree 10; it contains 994 such groups.Miller(1900b)gives what he considers a formal derivation of the primitive groups Of degree 10.

Cole(1895) determine the transitive groups of degree 12.His count for The primitives matches that of Sims(1970)' but for the imprimitives it differs from that of Royle(1987) in several places.Miller(1897a' 1897b' 18'8b and 1900a) determines the primitive groups of degree 13 to 17.His enumerations agree with those of sims(1970).In (1898a) he also determines the imprimitive groups of degree 14' obtaining 59. Miller(1898b) correctly gives a table of the numbers of soluble primitive groups degree up too 24.Bunside(1897) determines all primitive groups of degrees 3 to 8.His work is correct' except that he obtains two nonexistent primitive groups of degree 8' their construction having been left as an exercise.This error also ofcours in the second edition(1911) of his book' des Pite Miller(1899) having pointed it out.Murtin(1901) and kahn(1904) detremine the imprimitive groups of degree 15; they both find 70; Martin(1901) determines the primitive groups of degree 18; her list agrees with That of Sims.Bennett (1912) determines the primitive groups of degree 20; her list agrees with that of sims .

There dose not seem tobe any more literature on this topic until Sims (1970) lists all primitive groups of degrees up to 20.He says that he took the list from the literature and verified it partly by hand and partly by machine. He later extended this list to degree 50.Although the full list has never been published' it was made avilable inversion 3.5 of CAYLEY' released in 1987' as the CAYLEY library PRMGPS.

In Harada and yamaki(1979)' there is an undated reference to a master's thesis by Mizutani; the title of the thesis"The classification of primitive permutation groups of degree less than 49".No material from this thesis is seems to have been published. Pogorelov(1980 and 1982)lists' up to isomorphism' all primitive groups with insoluble for degree 21 to 64.This work was done by hand using M.Hall's classification of simple group of order less than 10^6 and various theorems on simple groups and primitive groups. Un fortunately he doesnot tabulate his results and so it is difficult to compare his list with that of Sims. in this paper we compute the JS –Maximal soluble subgroups of the group $GL(15,p^k)$.

2. Main Results

In this section we determine the JS –Maximal soluble subgoups of the group $GL(15, p^k)$.
For do this we say' by theorems $[2.5.4]$ and $[2.5.8]$ of $[16]$, we know that if M is a imprimitive maximal soluble subgroup of the group $GL(15, p^k)$ then we can write M as follows: $M := \mathrm{N}m(V_1)|_{V_1}$ wr $M\theta$ Where $Nm(V_1)|_{V_1}$ is a primitive maximal soluble subgroup of the group $GL(V_1)$ and $M\theta$ is a transitive maximal soluble subgroup of $Sym(B)$, that $B=\{V_1,..., V_m\}$ be an unrefinable system of imprimitivity for M and $\theta: M \to Sym(B)$ is the homomorphism difined by $\forall g \in M, V_i(g\theta) = Vig.$

And also by theorem $[2.5.9]$ of $[16]$ if m is a proper divisor of the number 15. Then every imprimitive maximal soluble subgroup of the group $GL(15, p^k)$ has experssion as follows : $M = P\ wr\ T$, where P is a primitive maximal soluble subgroup of the group $GL(m, p^k)$ and T is a transitive maximal soluble subgroup of the symetric group $S_{\frac{m}{15}}$.

Also by theorems $[2.5.35]$ and $[2.5.37]$ of $[16]$ We know that if M is a primitive maximal soluble subgroup of the group $GL(15, p^k)$, Whose the unique maximal abelian normal subgroup has order $p^{km}-1$.Then' we can write M as follows: $M := (C_{p^{km}-1} \mathrm{Y}\ E)\mathrm{N}\ D,$ Where E is extraspecial of order 5^3 and exponent 5 or 4 and D is a completely reducible (not maximal) soluble subgroup determined in theorems $[2.5.35]$ and $[2.5.37]$ of $[16]$.Now by using a bove notions' we prove the following Lemmas and theorem.

Lemma16:

Let p be the prime number and k,n be the positive integers and let F be the field of p^k element. Then the number types of JS -imprimitive maximal soluble subgroups in the group $GL(15, p^k)$ is 6 and are $M_i, i=1,..., 6.$

Proof:

Since by theorems $[2.1.3]$ and $[2.1.4]$ of $[16]$ every thransitive maximal soluble subgroup of the symetric group S_{15} is conjugate to $S_3\ wr\ Hol(C_5)$ or $Hol(C_5)\ wr\ S_3$.Therefore by using the JS -Maximmal soluble subgroups of $GL(3, p^k)$ and $GL(5, p^k)$, the JS -imprimitive maximal soluble subgroup of the $GL(15, p^k)$ are as follows:

$$M_1(15, p^k) := GL(1, p^k)wr(S_3 wr Hol(C_5)), (p^k \neq 2),$$

$$M_2(15, p^k) := GL(1, p^k)wr(Hol(C_5)wrS_3), (p^k \neq 2),$$

And Since the JS -imprimitive maximal soluble subgroups of a group are considering as $PwrT$, where P is a primitive maximal soluble subgroup of the group $GL(3, p^k)$ and T is a transitive maximal soluble subgroup of the symetric group S_5, namely $Hol(C_5)$.
Thus we have also:

$$M_3(15, p^k) := M_2(3, p^k)wr(Hol(C_5)), (p^k \neq 2),$$

$$M_4(15, p^k) := M_3(3, p^k)wr(Hol(C_5)), p^k \equiv 1(\mathrm{mod}3),$$

Now if we consider P is a primitive maximal soluble subgroup of the group $GL(5, p^k)$ and T is a transitive maximal soluble group of the symetric group S_2 .Then we have:

$$M_5(15, p^k) := M_2(5, p^k) wr S_3,$$
$$M_6(15, p^k) := M_3(5, p^k) wr S_3, p^k \equiv 1(\mathrm{mod}5),$$

And the proof of lemma is complete.

Lemma 17:

let p be prime number and k, n be positive integers and let F be the fild of p^k element.Then the number types of JS -primitive maximal soluble subgroups in the group $GL(15, p^k)$ is 4 and are $\mathrm{M}_i, i = 7, \ldots, 10$.

Proof:

By theorems $[2.5.35]$ and $[2.5.37]$ of $[16]$, the unique maximal abelian normal subgroups Of the a group $GL(15, p^k)$ has order $p^{15k} - 1, p^{5k} - 1, p^{3k} - 1$ or $p^k - 1$.

Therefore there is just one JS –primitive maximal soluble subgroup of order $p^{15k} - 1$, namely, $M_1(15, p^k) := C_{p^{15k}-1} \times C_{15}$, the normaliser of singer cycle.

And also by the same theorems the JS –primitive maximal soluble subgroups with unique maximal abelian normal subgroups has order $p^{5k} - 1$ are listed as follows :

$$M_8(15, p^k) := M_3(3, p^{5k}) \times C_5, p^{5k} \equiv 1(\mathrm{mod}3),$$

And the JS –primitive maximal soluble subgrups of the group $GL(15, p^k)$, whose unique maximal abelian normal subgrouphas order of $p^{3k} - 1$, are listed below:

$$M_9(15, p^k) := M_4(5, p^{3k}) \times C_3, p^{3k} \equiv (\mathrm{mod}5),$$

And also the JS –primitive maximal soluble subgroups of the group $GL(15, p^k)$, with the unique maximal abelian normal subgroup has order of $p^k - 1$, are listed as follows : $M_{10}(15, p^k) := M_3(3, p^k) \otimes M_3(5, p^k), p^k \equiv 1(\mathrm{mod}15).$

And the proof of lemma is complete.

Theorem 18(B.Razzaghmaneshi):

Let p be the prime number and k, n be positive integers sand let F be the field of p^k element.Then the number types of JS –maximal soluble subgroups in the group $GL(15, p^k)$ is 10 and are $M_i, i = 1, \ldots, 10$.

Proof :

Since the JS –Maximal soluble subgroups of the group $GL(15, p^k)$ is the same of JS -primitive and imprimitive maximal soluble subgroup S of the Group $GL(15, p^k)$.Thus by using of the lemmas 4 and 5 obviously that the Number types of JS -Maximal soluble subgroups of the group $GL(15, p^k)$ is 10 and are $M_i, i = 1, \ldots, 10$.
And the proof of theorem is complete.

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